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Mathematical Competencies: A Conceptual Framework for 21st-Century Education

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Abstract

Mathematical competencies have become an essential component of 21st-century education. These competencies foster critical thinking, problem-solving, reasoning, communication, and decision-making skills in learners, abilities that are crucial for academic success and lifelong learning. In today's rapidly evolving, technology-driven, and knowledge-based society, mathematics education extends beyond the mere acquisition of procedural knowledge; it emphasizes the meaningful application of mathematical understanding to diverse real-life situations. Consequently, education systems worldwide have increasingly adopted competency-based mathematics education as a key strategy to prepare learners for the social, economic, and technological challenges of the 21st century (OECD, 2019; UNESCO, 2015).

This conceptual paper aims to examine the nature and significance of mathematical competencies and to propose a conceptual framework that aligns these competencies with the goals of 21st-century education. The study employs a systematic conceptual literature review methodology, critically examining scholarly books, peer-reviewed journal articles, and international educational policy documents. Relevant literature was identified through recognized academic databases such as Scopus, ERIC, Web of Science, and Google Scholar, and analyzed using thematic analysis to identify recurring ideas and theoretical perspectives regarding mathematical competencies.

This review identifies eight key mathematical competencies as foundational for contemporary mathematics education: mathematical thinking, problem-solving, reasoning, symbols and rules, representation, communication, modelling, and the use of tools and resources. Collectively, these competencies help foster conceptual understanding, creativity,

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analytical thinking, and collaboration among learners, while also equipping them to solve real-world problems encountered in academic, professional, and everyday life. The paper further suggests that strengthening these competencies through competency-based mathematics education can better prepare learners for lifelong learning and active participation in the twenty-first century.

This study integrates key international perspectives on mathematical competencies and highlights their significance in contemporary educational practices, thereby contributing a novel, synthesized conceptual framework to the existing body of knowledge. Its findings offer valuable insights for curriculum developers, teacher educators, mathematics teachers, and policymakers aiming to enhance competency-based mathematics education.

Keywords: Mathematical Competencies, 21st-Century Education, Competency-Based Education, Mathematics Education, Mathematical Thinking.

Introduction

Mathematics has been recognized as a fundamental subject that helps individuals understand, interpret, and solve problems encountered in everyday life (Kilpatrick et al., 2001; NCTM, 2000). However, the demands of the twenty-first century have shifted the goals of mathematics education; the aim is no longer merely to acquire procedural knowledge but to develop broad mathematical competencies that foster critical thinking, problem-solving, reasoning, communication, and informed decision-making. As society becomes increasingly driven by knowledge and technology, learners are expected not only to master mathematical concepts but also to apply them effectively across diverse academic, professional, and real-life contexts. Consequently, mathematical competencies have emerged as essential foundational skills that prepare learners for lifelong learning and meaningful participation in contemporary society (Niss, 2003; OECD, 2019; UNESCO, 2015).

Recent educational reforms worldwide have emphasized 'competency-based education' to prepare learners for the challenges and opportunities of the twenty-first century (OECD, 2019; UNESCO, 2015). International frameworks such as the Organisation for Economic Cooperation and Development (OECD), Programfor International Student Assessment (PISA), and the National Council of Teachers of Mathematics (NCTM) stress that effective mathematics education should foster the ability to think and reason mathematically, communicate ideas, solve real-life problems, and apply mathematical knowledge in new or unfamiliar situations. Similarly, contemporary curriculum reforms promote a shift away from rote learning toward learning based on conceptual understanding, inquiry, and application; thus, mathematical competencies are placed at the core of high-quality mathematics education (NCTM, 2000; Kilpatrick et al., 2001; OECD, 2019).

While mathematical capabilities are increasingly being recognized, many education systems continue to prioritize exam performance and process-based learning over the development

of skills essential for lifelong learning. Although there is considerable literature on mathematical proficiency, mathematical literacy, and competency-based education, these concepts are often viewed in isolation, limiting conceptual coherence among them. Furthermore, few studies have attempted to integrate key international frameworks into a comprehensive conceptual framework capable of explaining the role of mathematical capabilities in twenty-first-century education (Boaler, 2016; Niss, 2003).

This paper addresses this gap by examining the concept of mathematical competencies through the lens of contemporary educational research and international competency frameworks. Synthesizing existing theoretical and empirical literature, the study proposes a conceptual framework that elucidates the nature, dimensions, and educational significance of mathematical competencies. The framework aims to inform curriculum development, teaching methodologies, assessment, and teacher training, while also contributing to the ongoing discourse on competency-based mathematics education.

Rapid advancements in science, technology, globalization, and digital transformation have significantly altered the knowledge and skills expected of learners in the twenty-first century. Mathematics education is no longer confined merely to accuracy in calculation; it is now also expected to foster higher-order thinking, creativity, collaboration, communication, and problem-solving abilities. These competencies enable learners to critically analyze information, make informed decisions, and apply mathematical knowledge across diverse real-life situations. Consequently, mathematical proficiency is now regarded as a fundamental pillar of 21stcentury education and lifelong learning (OECD, 2019; UNESCO, 2015).

Existing literature discusses mathematical literacy, mathematical proficiency, competency-based education, and 21stcentury skills in detail. However, these perspectives are often presented in isolation, and efforts to integrate them into a coherent conceptual framework have been limited. Furthermore, there have been relatively few conceptual studies systematically examining the interrelationships between major international frameworks of mathematical competencies and their impact on contemporary mathematics education. This gap highlights the need for a comprehensive conceptual synthesis that clarifies how mathematical competencies can support twenty-first-century educational goals (Niss, 2003; Kilpatrick et al., 2001).

Purpose of the Study:

This study aims to develop a conceptual understanding of mathematical competencies by integrating key theoretical perspectives in mathematics education with international competency-based frameworks. Specifically, the study seeks to examine the nature and dimensions of mathematical competencies, analyze their relationship with twenty-first-century education, and propose a conceptual framework capable of guiding curriculum development, classroom activities, assessment, and teacher training.

Significance of the Study:

This study contributes to the growing body of knowledge on competency-based mathematics education by providing a comprehensive conceptual framework for understanding mathematical competencies in the context of twenty-first-century education. It is anticipated that the proposed framework will assist curriculum developers in designing competency-oriented mathematics curricula, teachers in adopting learner-center pedagogical approaches, and teacher educators in preparing future mathematics teachers for competency-based instruction. Furthermore, the study offers a theoretical foundation for education policymakers and researchers to strengthen mathematics education and foster the development of competencies essential for lifelong learning and responsible participation in contemporary society.

Methodology:

Research Design

This study adopts a 'Systematic Conceptual Literature Review' (SCLR) as its research design to develop a comprehensive understanding of mathematical capabilities within the context of 21stcentury education. Unlike empirical studies, a conceptual literature review focuses on analyzing, synthesizing, and interpreting existing scholarly knowledge to construct a coherent theoretical framework. This review systematically examines published literature to identify key concepts, theoretical perspectives, and common themes related to mathematical capabilities. This approach enables the integration of diverse viewpoints and facilitates the development of a conceptual framework capable of guiding mathematics education, curriculum development, and educational practice (Snyder, 2019; Xiao & Watson, 2019).

Data Sources

To ensure the reliability and relevance of the sources used in this study, information was gathered from internationally recognized academic databases and documents related to education policy (Snyder, 2019; Xiao & Watson, 2019). Key databases included Scopus, ERIC, Web of Science, and Google Scholar. Additionally, significant international reports and policy documents from organizations such as the OECD, UNESCO, and the National Council of Teachers of Mathematics (NCTM) were consulted to gain theoretical and policy-oriented insights into mathematical competencies and competency-based education. Keywords such as 'mathematical competencies', 'mathematical proficiency', 'mathematical literacy', 'competency-based education', '21stcentury skills', and 'mathematics education' were used to search for information.

Data Analysis

The collected literature was analyzed using 'thematic analysis.' This is a qualitative method that helps identify and understand recurring concepts across various studies. The analysis

involved repeatedly reading the selected literature, coding key concepts, grouping related ideas into broader themes, and synthesizing these themes to construct an integrated conceptual framework. Particular emphasis was placed on identifying common aspects of mathematical competencies within major international frameworks, such as mathematical thinking, problem-solving, reasoning, communication, presentation, modeling, and the effective use of mathematical tools. The findings of the thematic analysis provided the foundation for developing the conceptual framework presented in this study (Braun & Clarke, 2006).

Theoretical Foundations of Mathematical Competencies

Understanding Mathematical Competencies

Mathematical competence refers to the knowledge, skills, attitudes, and dispositions that enable individuals to understand, apply, and effectively communicate mathematical ideas across various situations. These competencies extend far beyond the mere recall of facts or procedural mastery; they emphasize the ability to think mathematically, solve novel or unfamiliar problems, interpret quantitative information, and make informed decisions. Mathematical competence requires learners to connect their conceptual understanding with practical application, thereby enabling them to use mathematics meaningfully in academic, professional, and everyday contexts. Consequently, in contemporary mathematics education, mathematical competence is viewed as a crucial outcome of competency-based learning, rather than simply a matter of mastering mathematical content (Niss, 2003; Kilpatrick et al., 2001).

Mathematical proficiencies encompass high-level cognitive processes such as mathematical thinking, logical reasoning, representation, communication, modeling, and the strategic use of mathematical tools. These proficiencies help learners develop confidence, flexibility, and independence when solving mathematical problems. As education systems increasingly emphasize learner-centered instruction and practical application, mathematical proficiencies play a fundamental role in preparing learners for lifelong learning and responsible participation in society (NCTM, 2000; OECD, 2019).

Evolution of Mathematical Competencies

Over the past few decades, the concept of mathematical proficiency has evolved significantly due to shifting educational priorities and the growing demand for transferable skills, skills applicable across various contexts. Traditionally, mathematics education focused primarily on calculation accuracy, rote memorization of formulas, and procedural mastery(Kilpatrick et al., 2001; NCTM, 2000). Success in mathematics was often measured by a student's ability to replicate algorithms and arrive at correct answers in examinations(Kilpatrick et al., 2001; Cai et al., 2017). While procedural knowledge remains important, contemporary educational research recognizes that mathematics education should also foster conceptual understanding, reasoning abilities, problem-solving skills, and the capacity to apply mathematical knowledge to real-world situations (Kilpatrick et al., 2001).

International educational reforms significantly influenced the shift towards competency-based mathematics education; these reforms emphasized learning outcomes rather than mere content coverage. Frameworks developed by researchers and organizations, such as Niss, the OECD/PISA program, and the National Council of Teachers of Mathematics, broadened the understanding of mathematics learning by highlighting the competencies essential for effective participation in modern society. Consequently, mathematical competencies are now viewed as integrated capabilities combining knowledge, skills, attitudes, and practical application, thereby making mathematics education more relevant to the needs of the twenty-first century (Niss, 2003; OECD, 2019).

Mathematical Competencies in 21stCentury Education

In twenty-first-century education, learners need to develop capabilities that enable them to adapt to rapid technological changes, global interconnectedness, and increasingly complex social and economic environments (Trilling & Fadel, 2009; OECD, 2019). In this context, mathematical competencies play a crucial role by fostering critical thinking, analytical reasoning, creativity, communication, collaboration, and the ability to make evidence-based decisions. Rather than viewing mathematics merely as an academic subject, contemporary education regards it as a means to develop skills that facilitate lifelong learning and success across various fields (OECD, 2019; UNESCO, 2015).

Competency-based mathematics education encourages learners to investigate problems, construct mathematical arguments, analyze data, evaluate solutions, and apply mathematical knowledge in unfamiliar situations. Such learning experiences help students become active participants in the learning process while also boosting their confidence and self-reliance. Therefore, mathematical capabilities are increasingly being recognized as essential core competencies for achieving the broad educational goals of twenty-first-century education and for preparing learners for future academic, professional, and societal challenges (NCTM, 2000; Boaler, 2016).

Major International Frameworks of Mathematical Competencies

Niss's Mathematical Competencies

Niss's framework is one of the most comprehensive and influential conceptualizations of mathematical competencies. Developed through the Danish KOM project, this framework identifies eight interconnected mathematical competencies that learners need to engage successfully with mathematics in both academic and real-life contexts. These competencies include mathematical thinking, problem-solving, modeling, reasoning, representation, symbols and formalism, communication, and the use of mathematical aids and tools. Rather than viewing mathematics merely as the rote memorization of rules and procedures, Niss emphasizes the learner's ability to understand, apply, and effectively express mathematical ideas. This framework has significantly influenced competency-based mathematics curricula and educational research at an international level (Niss, 2003).

OECD/PISA Mathematical Literacy

The OECD's 'Program for International Student Assessment' (PISA) views mathematical ability through the lens of 'mathematical literacy.' Mathematical literacy refers to an individual's capacity to understand, use, and interpret mathematics in various real-life situations. It extends far beyond classroom success; it emphasizes reasoning, problem-solving, communication, and making informed decisions. According to the OECD, mathematically literate individuals can recognize the role of mathematics in society and apply mathematical knowledge to address practical challenges. This perspective has shaped mathematics education policies worldwide by encouraging schools to prioritize capacity building alongside conceptual understanding (OECD, 2022; OECD, 2023).

Kilpatrick et al.'s Mathematical Proficiency

Kilpatrick, Swafford, and Findell (2001) introduced the concept of mathematical proficiency, comprising five interconnected strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Collectively, these dimensions describe the knowledge, skills, reasoning abilities, and positive attitudes essential for successful mathematical learning. The framework highlights that effective mathematics education must balance procedural competence with conceptual understanding while fostering learners' confidence and willingness to engage with mathematical tasks. Mathematical proficiency has become a widely accepted theoretical foundation for curriculum design, assessment, and mathematics instruction.

NCTM Process Standards

The National Council of Teachers of Mathematics (NCTM) emphasizes five process standards for learning mathematics: problem-solving, reasoning and proof, communication, connections, and representation. These standards help develop learners' mathematical understanding through active participation, inquiry, and the meaningful application of knowledge. The NCTM framework encourages teachers to create a learning environment where students can articulate their thinking, justify solutions, exchange mathematical ideas, and establish connections between concepts and real-life experiences. These process standards remain a crucial foundation for competency-based mathematics education in many educational systems (NCTM, 2000).

Literature Review

The approach to mathematics education has evolved in recent times(Boaler, 2016; Gravemeijer et al., 2017). While the focus was previously on procedural knowledge and calculation accuracy, the emphasis has now shifted toward developing mathematical competencies, enabling learners to apply mathematics effectively across diverse situations. This shift signifies that learning mathematics is not merely about acquiring mathematical knowledge but also about developing the ability to reason, communicate, represent ideas,

and solve complex problems. Experts suggest that mathematical competencies equip learners to think critically, make sound decisions, and effectively utilize mathematical knowledge in academics, the workplace, and daily life. Consequently, competency-based mathematics education has become a key objective in educational reforms across many countries, with a heightened emphasis on conceptual understanding, application, and learner-centered instruction (Niss, 2003; Kilpatrick et al., 2001; NCTM, 2000).

Recent studies also indicate that mathematical competencies play a crucial role in students' self-confidence, creativity, and capacity for independent learning. Rather than viewing mathematics merely as a collection of procedures, contemporary approaches encourage learners to investigate problems, justify their reasoning, comprehend mathematical information, and articulate their ideas clearly. Such activities foster meaningful learning and strengthen the ability to apply mathematics in new situations, thereby making mathematical competencies an essential component of quality mathematics education (Boaler, 2016; OECD, 2022).

The growing demand for twenty-first-century skills has further heightened the importance of mathematical competencies in modern education systems. Skills such as critical thinking, problem-solving, communication, creativity, collaboration, and decision-making are now considered essential for lifelong learning and for successfully participating in an increasingly complex, technology-driven society. Mathematical competencies contribute directly to the development of these skills; they enable learners to analyze information, evaluate evidence, recognize patterns, and devise logical solutions to real-world problems (OECD, 2019; UNESCO, 2015).

Research indicates that competency-based mathematics education provides authentic learning experiences that encourage learners to apply mathematical knowledge in practical situations, rather than simply rote-learning procedures. Through activities involving mathematical reasoning, modeling, data interpretation, and communication, students develop competencies that extend beyond the mathematics classroom. Consequently, mathematical competencies are increasingly viewed as core capabilities that support the broader educational goals of twenty-first-century education and lifelong learning (NCTM, 2000; Boaler, 2016).

Mathematics education in schools plays a crucial role in developing students' mathematical capabilities. It provides opportunities for them to develop an understanding of concepts, procedural fluency, and skills in reasoning and problem-solving. Contemporary mathematics curricula promote inquiry-based, activity-centered, and competency-based learning approaches, engaging students in meaningful mathematical experiences (Common Core State Standards Initiative, 2010; NCTM, 2014). Rather than focusing solely on obtaining correct answers, effective mathematics education encourages students to articulate their thinking, justify their solutions, exchange ideas, and connect mathematical concepts to real-life situations (Kilpatrick et al., 2001; NCTM, 2000).

Studies have also shown that classroom practices significantly influence the development of mathematical abilities (Darling-Hammond et al., 2020; Cai et al., 2017). A learning environment that fosters inquiry, discussion, collaboration, and reflection boosts students' confidence and deepens their understanding of mathematical concepts. Conversely, teaching methods that prioritize rote memorization and a narrow focus on examinations often limit opportunities for learners to develop high-level mathematical capabilities. Therefore, strengthening competency-based school mathematics is essential to prepare learners for the educational and professional demands of the twenty-first century (Boaler, 2016; OECD, 2022).

Research Gap

Although there has been considerable research on mathematical proficiency, mathematical literacy, competency-based education, and 21stcentury skills, these areas are often viewed as distinct topics. Existing studies generally focus on individual competency frameworks or specific educational contexts; relatively little attention has been paid to integrating key international perspectives to develop a unified conceptual understanding of mathematical competencies. Furthermore, many studies place greater emphasis on curriculum implementation and classroom activities rather than on a comprehensive theoretical analysis of the competencies essential for 21stcentury mathematics education (Niss, 2003; OECD, 2022).

This review highlights the need for a conceptual framework that brings together influential international perspectives on mathematical competencies and elucidates their collective contribution to contemporary mathematics education. Addressing this gap, the study synthesizes key theoretical frameworks to develop an integrated conceptual framework. This framework can inform curriculum development, teaching methodologies, assessment, teacher training, and future research in competency-based mathematics education.

Mathematical Competencies: A Conceptual Framework for 21stCentury Education

Mathematical Thinking Competence: Mathematical thinking capability refers to an individual's ability to explore, analyze, generalize, and organize mathematical ideas in a logical and systematic manner. It enables learners to recognize patterns, formulate mathematical questions, make conjectures, and employ abstract thinking to solve both familiar and novel problems. Rather than relying on rote methods, mathematically competent learners construct their own understanding by connecting concepts and reasoning through mathematical situations. This capability forms the foundation for higher-order thinking and assists learners in developing analytical and reflective approaches to problem-solving (Niss, 2003; Schoenfeld, 2016).

In the context of 21stcentury education, mathematical thinking capability contributes to critical thinking, creativity, innovation, and independent learning. Learners who develop mathematical thinking become capable of analyzing complex situations, evaluating

evidence, and making informed decisions, competencies that are essential in contemporary education and professional life (OECD, 2019).

Problem Tackling Competence: Problem-solving ability entails identifying, understanding, and solving mathematical problems by selecting appropriate methods and evaluating potential solutions. It involves grasping the nature of the problem, planning solution strategies, applying correct mathematical techniques, and reflecting on the effectiveness of the results obtained. This ability encourages learners to confront new situations with confidence and a systematic approach, rather than relying solely on commonly used algorithms (Niss, 2003).

Problem-solving is considered one of the most essential competencies in 21st-century education, as modern society demands individuals capable of addressing complex, real-world challenges. Consequently, competency-based mathematics education encourages students to investigate real-world problems, collaborate with others, and develop flexible solution strategies applicable across diverse contexts (OECD, 2022; NCTM, 2000).

Reasoning Competence: Reasoning ability entails thinking logically, justifying mathematical arguments, evaluating evidence, and drawing valid conclusions based on mathematical principles. It helps learners understand how mathematical processes work, enables them to compare different solution methods, and empowers them to construct sound mathematical arguments. Logical reasoning fosters conceptual understanding and boosts learners' confidence in making mathematical decisions (Kilpatrick et al., 2001).

Reasoning ability is deeply linked to critical thinking and evidence-based decision-making, two essential characteristics of 21stcentury education. Learners who develop strong reasoning skills become capable of analyzing information objectively, evaluating alternative perspectives, and solving problems through logical reasoning rather than rote memorization (NCTM, 2000; OECD, 2019).

Symbols and Formalism Competence: Proficiency in symbols and formalism refers to the ability to understand, interpret, manipulate, and correctly use mathematical symbols, expressions, formulas, and formal language. Mathematical symbols provide a universal language through which mathematical relationships and ideas can be conveyed precisely and efficiently. Proficient learners not only understand the procedures associated with symbolic methods but also grasp the concepts and meanings underlying mathematical notation (Niss, 2003).

In contemporary education, this competence enables learners to work effectively with mathematical models, algorithms, and technical applications. As digital technologies increasingly rely on symbols and formal mathematical methods, mastering this competence has become essential for advanced studies in fields such as science, engineering, economics, and technology (OECD, 2022).

Representational Competence: Representational competence (i.e., the ability to represent information in various ways) refers to the capacity to understand, construct, and transform mathematical ideas across different forms; this includes methods such as graphs, tables, diagrams, equations, charts, and visual models. These diverse modes of representation help learners grasp mathematical concepts from multiple perspectives and develop a deeper understanding. Learners possessing this competence can navigate easily and flexibly between visual, symbolic, verbal, and numerical forms of mathematical information (NCTM, 2000).

In 21stcentury education, this competence facilitates data interpretation, visual communication, digital literacy, and interdisciplinary learning. It enables learners to organize information effectively and communicate mathematical ideas in ways that enhance understanding and problem-solving capabilities (Boaler, 2016).

Communicative Competence: Communicative competence refers to the ability to express, explain, discuss, and justify mathematical ideas using appropriate mathematical language, symbols, diagrams, and logical arguments. Learning mathematics becomes more meaningful when learners actively articulate their thinking, defend their solutions, and engage in discussions with peers and teachers. Effective mathematical communication strengthens conceptual understanding and fosters reflective learning (NCTM, 2000).

Communication is considered one of the essential competencies for 21stcentury education, as it enables learners to collaborate effectively, share knowledge, and solve problems together. Therefore, mathematical communication contributes not only to academic achievement but also to teamwork, leadership, and professional success (OECD, 2019).

Modelling Competence: Modeling competence entails creating mathematical models that represent real-world situations, analyzing these models using mathematical methods, interpreting the results, and evaluating their suitability. Mathematical modeling bridges the gap between theoretical mathematics and practical application; it assists learners in solving real-world problems encountered in science, economics, engineering, business, and daily life (Niss, 2003).

This competence has become crucial in 21stcentury education because modern society requires individuals capable of analyzing complex systems, interpreting quantitative information, and developing evidence-based solutions for practical challenges. Consequently, modeling competence fosters interdisciplinary learning and enhances learners' ability to apply mathematics beyond the classroom (OECD, 2022).

Aids and Tools Competence: Proficiency in using aids and tools entails the effective use of mathematical instruments, calculators, digital technology, statistical software, spreadsheets, and other technical resources to explore mathematical problems and derive solutions. Proficient learners understand when technical tools are appropriate and how they can enhance mathematical inquiry without compromising the understanding of underlying concepts (Niss, 2003).

Digital technology has become an integral part of 21stcentury education. Consequently, learners are expected to use technical tools responsibly to analyze data, visualize mathematical relationships, run simulations, and solve complex problems. The effective integration of technical tools enhances the mathematical learning process while simultaneously strengthening digital proficiency and lifelong learning skills (OECD, 2019).

Integration of Mathematical Competencies for 21st-Century Education

Mathematical competencies should be viewed not as isolated skills, but as interconnected facets of effective mathematical learning(Kilpatrick et al., 2001; Blömeke et al., 2015). Mathematical thinking fosters reasoning; reasoning capabilities strengthen problem-solving; representation enhances communication; modeling links mathematics to real-world situations; and technological tools facilitate exploration and analysis. Together, these competencies form an integrated framework that helps develop conceptual understanding, critical thinking, creativity, collaboration, and informed decision-making skills in learners.

This conceptual framework demonstrates that mathematical competencies are essential for 21stcentury education. Integrating them into mathematics curricula, teaching methods, and assessment systems can prepare learners for lifelong learning, responsible citizenship, and participation in rapidly evolving knowledge-based and technology-driven societies. Therefore, competency-based mathematics education should focus on developing these interconnected competencies through meaningful, inquiry-based, and practical learning experiences, rather than merely emphasizing procedural knowledge (Niss, 2003; Kilpatrick et al., 2001; OECD, 2019).

Discussion

Mathematical Competencies and 21st-Century Skills

The findings of this conceptual review indicate that mathematical competencies are deeply intertwined with the knowledge, skills, and attitudes essential for 21stcentury education. Contemporary educational frameworks consistently emphasize the need for learners to develop capabilities such as critical thinking, creativity, problem-solving, communication, collaboration, and informed decision-making to effectively navigate the challenges of a rapidly changing world(Voogt & Roblin, 2012; UNESCO, 2021). Mathematical competencies provide a crucial foundation for these broader educational goals by enabling learners to analyze information, think logically, interpret quantitative evidence, and apply mathematical knowledge in real-world contexts. Therefore, mathematics education should move beyond mere procedural learning to focus on cultivating competencies that prepare learners for lifelong learning and responsible participation in society (OECD, 2019; UNESCO, 2015).

This review also reveals that major international frameworks, such as those proposed by Niss (2003), Kilpatrick et al. (2001), OECD/PISA, and the National Council of Teachers of

Mathematics, place a consistent emphasis on meaningful mathematical learning. Although these frameworks differ in terminology and structure, they collectively highlight mathematical thinking, reasoning, problem-solving, communication, representation, and application as essential competencies for effective mathematics education. This convergence suggests that mathematical competencies should be viewed as integrated capabilities rather than isolated skills. Developing these competencies not only enables learners to achieve academic success but also helps them adapt to technological advancements, participate in evidence-based decision-making, and solve complex real-world problems. Therefore, mathematical competencies constitute a fundamental component of competency-based education and an essential pillar of 21st-century education (Niss, 2003; NCTM, 2000; OECD, 2022).

Implications for Curriculum, Teaching, Assessment, and Teacher Education

The findings of this study are highly significant for curriculum development, classroom instruction, assessment methods, and teacher education. Mathematics curricula should foster conceptual understanding, mathematical thinking, problem-solving, communication, and modeling, rather than focusing solely on procedural knowledge and exam performance. A competency-based curriculum should provide learners with authentic mathematical experiences that encourage inquiry, collaboration, and the application of mathematics to real-life situations. Such an approach can enhance learners' confidence, motivation, and ability to apply mathematical knowledge across diverse contexts (NCTM, 2000; OECD, 2019).

Similarly, teaching methods should shift towards learner-centered approaches that actively engage students in mathematical exploration, discussion, inquiry, and reflection(Darling-Hammond et al., 2020; NCTM, 2014). Teachers should create a classroom environment that encourages learners to justify their reasoning, articulate mathematical ideas, examine alternative solutions, and effectively use technological tools. Assessment methods must also align with competency-based education; rather than relying solely on traditional written examinations, assessments should evaluate learners' conceptual understanding, reasoning processes, communication skills, and problem-solving abilities. Furthermore, teacher training programs should equip prospective and practicing teachers with the pedagogical knowledge and professional competencies required to effectively implement competency-based mathematics education. Strengthening curriculum, instruction, assessment, and teacher preparation through a competency-based approach will help enhance the quality and relevance of mathematics education in the twenty-first century (Kilpatrick et al., 2001; Boaler, 2016; OECD, 2022).

Conclusion

Mathematical competencies have become a core component of competency-based mathematics education and an essential foundation for achieving the goals of 21stcentury

education. This conceptual review suggests that contemporary mathematics education should not be limited merely to procedural knowledge and computational skills; rather, it should foster competencies such as mathematical thinking, problem-solving, reasoning, communication, presentation, modeling, and the effective use of mathematical tools. These competencies enable learners to apply mathematical knowledge in meaningful contexts, make informed decisions, and tackle complex challenges encountered in academic, professional, and everyday life (Niss, 2003; Kilpatrick et al., 2001).

The review also reveals that major international frameworks, including those by Niss, OECD/PISA, Kilpatrick, and others, as well as the framework proposed by the National Council of Teachers of Mathematics, collectively emphasize the importance of competency-based mathematics education. Although these frameworks differ in structure and terminology, they share a common objective: to prepare learners to use mathematics effectively through conceptual understanding, critical thinking, and real-world application. The conceptual framework proposed in this study integrates these complementary perspectives and highlights the interconnected nature of mathematical competencies in contemporary education (NCTM, 2000; OECD, 2022).

This study concludes that to enhance the quality and relevance of mathematics education, strengthening mathematical competencies should be a primary priority for curriculum developers, teachers, teacher educators, and policymakers. A competency-based approach, which fosters inquiry, reasoning, collaboration, communication, and real-world problem-solving, can better prepare learners for lifelong learning and active participation in a knowledge-based society(UNESCO, 2021; OECD, 2023). Future research could further validate and refine the proposed conceptual framework through empirical studies conducted across diverse educational contexts, thereby contributing to the continued advancement of competency-based mathematics education (Boaler, 2016; UNESCO, 2015).

Future Research

Future research should focus on empirically verifying the proposed conceptual framework across various educational levels and contexts. Qualitative and quantitative studies could explore how mathematical competencies influence students' academic achievement, problem-solving abilities, critical thinking, and readiness for lifelong learning. Comparative studies of different national curriculum frameworks could also provide deeper insights into the implementation of competency-based mathematics education. Furthermore, future research could examine effective teaching strategies, assessment methods, and teacher professional development programs that foster the development of mathematical competencies in 21st-century classrooms. Such inquiries would contribute to strengthening the theoretical and practical foundations of competency-based mathematics education (OECD, 2022; Boaler, 2016).

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389–407.

Blömeke, S., Gustafsson, J. E., & Shavelson, R. J. (2015). Beyond dichotomies: Competence viewed as a continuum. Zeitschrift für Psychologie, 223(1), 3–13.

Boaler, J. (2016). Mathematical mindsets: Unleashing students' potential through creative mathematics, inspiring messages and innovative teaching. Jossey-Bass.

Boaler, J. (2022). Mathematical mindsets (2nd ed.). Jossey-Bass.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. Qualitative Research in Psychology, 3(2), 77–101.

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., & Hiebert, J. (2017). The role of conceptual understanding in mathematics education. Journal for Research in Mathematics Education, 48(2), 118–123.

Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics.

National Governors Association Center for Best Practices & Council of Chief State School Officers.

Darling-Hammond, L., Flook, L., Cook-Harvey, C., Barron, B., & Osher, D. (2020). Implications for educational practice of the science of learning and development. Applied Developmental Science, 24(2), 97–140.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? International Journal of Science and Mathematics Education, 15(Suppl. 1), 105–123.

Hoyles, C., & Lagrange, J. B. (Eds.). (2010). Mathematics education and technology: Rethinking the terrain. Springer.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. National Academy Press.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics.

National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. National Council of Teachers of Mathematics.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. Roskilde University.

OECD. (2019). OECD Future of Education and Skills 2030: OECD Learning Compass 2030. OECD Publishing.

OECD. (2020). Education at a glance 2020: OECD indicators. OECD Publishing.

OECD. (2022). PISA 2022 Mathematics Framework. OECD Publishing.

OECD. (2023). PISA 2022 Results (Volume I): The State of Learning and Equity in Education. OECD Publishing.

Partnership for 21st Century Learning. (2019). Framework for 21st Century Learning. Battelle for Kids.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. Journal of Education, 196(2), 1–38.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2018). Teaching mathematics: Foundations to middle years. Oxford University Press.

Snyder, H. (2019). Literature review as a research methodology: An overview and guidelines. Journal of Business Research, 104, 333–339.

Trilling, B., & Fadel, C. (2009). 21st century skills: Learning for life in our times. Jossey-Bass.

UNESCO. (2015). Education 2030: Incheon declaration and framework for action. UNESCO.

UNESCO. (2017). A guide for ensuring inclusion and equity in education. UNESCO.

UNESCO. (2021). Reimagining our futures together: A new social contract for education. UNESCO.

Ünal, Z. E., Kartal, G., Ulusoy, S., Ala, A. M., Yilmaz, M. Z., & Geary, D. C. (2023). Relative contributions of g and basic domain-specific mathematics skills to complex mathematics competencies. Intelligence, 101, 101797.

Cevikbas, M., Kaiser, G., &Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. Educational Studies in Mathematics, 109(2), 205–236.

Voogt, J., & Roblin, N. P. (2012). A comparative analysis of international frameworks for 21st-century competences. Journal of Curriculum Studies, 44(3), 299–321.

Xiao, Y., & Watson, M. (2019). Guidance on conducting a systematic literature review. Journal of Planning Education and Research, 39(1), 93–112.